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Ground state of the two-dimensional isotropic antiferromagnetic Heisenberg model of $S = \frac{1}{2}$ on a square lattice

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Abstract. An application of the method developed in our previous papers to a two-dimensional isotropic Heisenberg antiferromagnet of $S = \frac{1}{2}$ on a square lattice is presented. A self-consistent independent spin-wave representation (SCISWR) is proposed in which we have performed the renormalization from both the dynamic and kinematics interaction and calculated the corrections from the correlation of the nearest-neighbour and next-nearest-neighbour sites. An anisotropy spin-wave excitation energy in unrestricted space is found self-consistently and has a gap. The difficulty of divergence of higher-order terms in usual spin-wave theory seems to have been overcome. Our calculation shows the ground-state energy $E \approx -0.659NJ$.

The spin-wave theory (SWT) has a long history and has worked well for three-dimensional magnetic systems. In the lower-dimensional case, the Mermin–Wagner theorem has prohibited any long-range magnetic order at finite temperature for isotropic magnetic Heisenberg systems. The usual SWT has met serious difficulty for low-dimensional systems due to the divergence in the higher-order corrections. The two-dimensional (2D) Heisenberg model has attracted considerable attention in recent years, perhaps due to the discovery of high-temperature superconductivity in some doped oxide compounds which is thought to be an $S = \frac{1}{2}$ antiferromagnet at zero doping, and has been shown to be evidence of long-range antiferromagnetic correlation at low temperatures. Many methods have been proposed [1–10] to deal with the $S = \frac{1}{2}$ isotropic antiferromagnetic Heisenberg model (AFHM) on a 2D square lattice. From a different theoretical scheme based on the slave boson of RVB [8] or the Schwinger boson [9], one can obtain different results. The numerical calculations [1–6] have supported the existence of long-range order at zero temperature and Green-function Monte Carlo [5] (size: 12×12) and projection Monte Carlo studies [6] (size: 32×32) give the ground-state energy as $E \approx -0.6692NJ$ and $-0.6698NJ$. Although free spin-wave approximation can work well in this case, the usual SWT [11, 12] has a divergence problem in the higher-order approximation even for the ground state of a 2D square lattice with long-range order. Takahashi [10] proposed a variation method, the 'modified SWT', in which the Mermin–Wagner theorem is treated as a constraint condition ($\langle S_i^z \rangle = 0$). It seems to show how to apply the SWT to a low-dimensional isotropic quantum magnetic system. Unfortunately, although their trick works well at finite temperature, we believe that it may not work at zero temperature, since the magnetization of sublattice $\langle S^z \rangle$ is not zero and, therefore, the constraint $\langle S^z = 0 \rangle$ in their paper will fail. In this paper, we will apply a complete Bose transformation [13–14] to the 2D AFHM and give the calculation for the ground-state energy.

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The Hamiltonian of the AFHM we will consider is

$$H = J_z \sum_{f,g} S_f^z S_g^z + \tfrac{1}{2} J_{xy} \sum_{f,g} [S_f^+ S_g^- + S_f^- S_g^+] \tag{1}$$

where J_z and J_{xy} are positive and equal to J for an isotropic system. An AFHM square lattice can be divided into two sublattices F and G. In usual SWT sublattices, one maps the spin operators into Bose operators by means of Holstein–Primakoff [15] or Dyson–Maleev [16, 17] transformations. However, the transformations only keep the commutations but not the physical space, so one must consider how to estimate such an additional kinematics interaction. The physical space of the boson representation must be restricted in proper space where the maximum number of spin excitations must not exceed $2S + 1$ since the number of eigenstates of S^z is $2S + 1$. This is a physical constraint of the spin wave described by boson excitation. In this paper, some projection operators have been introduced to keep this Bose transformation constraint.

Following [13, 14], we define a projection operator for each site from the definition of the step function $\theta(x)$. The projection operators were proved to have the following representation [13, 14]:

$$\theta_i = \theta(2S - a_i^\dagger a_i) = \sum_{l=0}^{\infty} B_l a_i^{\dagger l} a_i^l \qquad i \in F, G \tag{2}$$

where $B_0 = 1$, $B_l = 0$ $(l = 1, \ldots 2S)$ and

$$B_l = \frac{(-1)^{l-2S}(l-1)!}{(2S)!l!(l-2S-1)!} \qquad l \geqslant 2S+1. \tag{3}$$

In the case of $S = \frac{1}{2}$, we have $B_l = (-1)^{l-1}(l-1)/l!$ for $l \geqslant 0$. In this paper, we will only study the case where $S = \frac{1}{2}$. It is easy to extend the formalism to the general case.

For an antiferromagnetic system on a square lattice, the local Bose transformation is

$$S_f^- = \tilde{a}_f^\dagger = \theta_f a_f^\dagger \qquad S_f^+ = \tilde{a}_f = a_f \theta_f \tag{4}$$

$$S_f^z = (\tfrac{1}{2} - a_f^\dagger a_f)\theta_f = \tfrac{1}{2}\theta_f - \tilde{a}_f^\dagger \tilde{a}_f \qquad f \in F \tag{5}$$

$$S_g^+ = -\tilde{b}_g^\dagger = -\theta_g b_g^\dagger \qquad S_g^- = -\tilde{b}_g = -b_g \theta_g \tag{6}$$

$$S_g^z = -\tfrac{1}{2}\theta_g + \tilde{b}_g^\dagger \tilde{b}_g \qquad g \in G. \tag{7}$$

The operators $(a, a^\dagger, b, b^\dagger)$ satisfy the well known boson commutation.

Substituting expansion (2) of θ into (4)–(7), we obtain

$$S_f^- = \tilde{a}_f^\dagger = \sum_{l=0}^{\infty} C_{l+1} a_f^{\dagger(l+1)} a_f^l \tag{8}$$

$$S_f^z = \tfrac{1}{2}\left(1 - \sum_{l=0}^{\infty} D_l a_f^{\dagger l} a_f^l\right) \tag{9}$$

$$C_{l+1} = (-1)^l/l! \qquad D_l = (-1)^{l+1}(l+1)/l!. \tag{10}$$

The transformation of $S_f^\dagger$ is the conjugate of equation (8). We can also find the transformation for the sublattices G similarly.

$$S_g^- = -\tilde{b}_g^\dagger = -\sum_{l=0}^{\infty} C_{l+1} b_g^{\dagger(l+1)} b_g^l \tag{11}$$

$$S_g^z = -\tfrac{1}{2}\left(1 - \sum_{l=0}^{\infty} D_l b_g^{\dagger l} b_g^l\right). \tag{12}$$

For a system with N sites, we define a total projection operator P which projects any vector of state into the following physical proper space of N sites:

$$\begin{aligned} P &= \Pi_i^N \theta_i \\ &= \Theta_i \cdot \theta_i \\ &= \Theta_{ij}\theta_i\theta_j \end{aligned} \tag{13}$$

$$\Theta_i = \Pi_{k\neq i}\theta_k \tag{14}$$

$$\Theta_{ij} = \Pi_{k\neq ij}\theta_k. \tag{15}$$

The final Bose transformation should be

$$\tilde{S}_f^- = \Theta_f \tilde{a}_f^\dagger \qquad \tilde{S}_g^\dagger = \Theta_g \tilde{b}_g^\dagger \tag{16}$$

$$\tilde{S}_f^z = \tfrac{1}{2}\Theta_f\left(1 - \sum_{l=0}^{\infty} D_l a_f^{\dagger l} a_f^l\right) \tag{17}$$

$$\tilde{S}_g^z = -\tfrac{1}{2}\Theta_g\left(1 - \sum_{l=0}^{\infty} D_l b_g^{\dagger l} b_g^l\right). \tag{18}$$

The constraint on the spin-wave excitation in the transformation has been included automatically. The transformed Heisenberg antiferromagnetic Hamiltonian is

$$\tilde{H} = J_z \sum_{f,g} \tilde{S}_f^z \tilde{S}_g^z - \tfrac{1}{2} J_{xy} \sum_{f,g} (\tilde{S}_f^+ \tilde{S}_g^- + \tilde{S}_f^- \tilde{S}_g^+). \tag{19}$$

In [14] it was proved that the following model Hamiltonian H has the same eigenvalues as $\tilde{H}$:

$$H = J_z \sum_{f,g} S_f^z S_g^z - \tfrac{1}{2} J_{xy} \sum_{f,g} (\tilde{a}_f \tilde{b}_g + \tilde{a}_f^\dagger \tilde{b}_g^\dagger) \tag{20}$$

where the operator S_f^z, S_g^z is defined by equations (5) and (7). First, we will find a self-consistent independent spin-wave representation (SCISWR) as a non-interacting part of a Hamiltonian and assume that

$$H = U_0 + H_2(\eta) + H_I(\eta) \tag{21}$$

$$H_2(\eta) = \tfrac{1}{2} J_z' \left[\sum_{f,g} (a_f^\dagger a_f + b_g^\dagger b_g) - \eta \sum_{f,g} (a_f b_g + a_f^\dagger b_g^\dagger)\right] \tag{22}$$

where $H_I(\eta)(= H - H_2(\eta))$ is the remainder interaction of spin waves and the parameters J_z' and η will be determined self-consistently. However, η is always one and $J_z' = J_z$ for an isotropic system in usual SWT. The $H_2(\eta)$ will be considered as a free part of the Hamiltonian and can be diagonalized by means of a Bogolubov transformation. The excitation energy of free spin waves in the representation of $H_2(\eta)$ is

$$\varepsilon_k = \tfrac{1}{2} J_z' \sqrt{1 - \eta^2 \gamma_k^2} \tag{23}$$

$$\gamma_k = \tfrac{1}{2}(\cos k_x d + \cos k_y d) \tag{24}$$

where J_z' is the renormalized parameter of interaction and d is the distance between the nearest-neighbour sites. If the self-consistent solution of η is not one, it means that the excitation of the renormalized spin wave in unrestricted Bose space is anisotropic. This representation is named the SCISWR.

In the SCISWR, the averages $\langle a_f^\dagger b_g \rangle$, $\langle a_f b_g^\dagger \rangle$, $\langle a_f^\dagger a_{f'}^\dagger \rangle$, $\langle a_f a_{f'} \rangle$, $\langle b_g^\dagger b_{g'}^\dagger \rangle$ and $\langle b_g b_{g'} \rangle$ must be zero and

$$\langle a_f^\dagger a_{f'} \rangle_\eta = f(f - f') - \tfrac{1}{2}\delta_{f,f'} \tag{25}$$

$$\langle b_g^\dagger b_{g'} \rangle_\eta = f(g - g') - \tfrac{1}{2}\delta_{g,g'} \tag{26}$$

$$n = \langle a_f^\dagger a_f \rangle_\eta = \langle b_g^\dagger b_g \rangle_\eta \tag{27}$$

$$\langle a_f^\dagger b_g^\dagger \rangle_\eta = \langle a_f b_g \rangle_\eta = g(f - g) \tag{28}$$

$$f(r) = \frac{1}{N} \sum_k \frac{1}{\sqrt{1 - \eta^2 \gamma_k^2}} \exp(-k \cdot r) \tag{29}$$

$$g = g(d) = \frac{1}{N} \sum_k \frac{\eta \gamma_k}{\sqrt{1 - \eta^2 \gamma_k^2}} \exp(-k \cdot d). \tag{30}$$

The SCISWR can be chosen so that the first perturbation contribution of the remainder interaction, $\langle H_I(\eta) \rangle_\eta$, is zero. In fact, such an approximation is equivalent to the following:

$$\begin{aligned} a_f^{\dagger k} a_f^k &\approx k^2 \langle a_f^{\dagger k-1} a_f^{k-1} \rangle_\eta a_f^\dagger a_f \\ &= k \cdot k! \langle a_f^\dagger a_f \rangle_\eta^{k-1} a_f^\dagger a_f = k \cdot k! n^{k-1} a_f^\dagger a_f \\ a_f^k b_g^k &\approx k^2 \langle a_f^{k-1} b_g^{k-1} \rangle_\eta a_f b_g \\ &= k \cdot k! \langle a_f b_g \rangle_\eta^{k-1} a_f b_g = k \cdot k! g(f - g)^{k-1} a_f b_g. \end{aligned} \tag{31}$$

Therefore, the approximation of independent spin wave representation is equivalent to

$$\begin{aligned} H \approx \frac{1}{2} \left[J_z \frac{(1 + 2x)}{(1 - x)^4 (1 + n)^5} + 4 J_{xy} \frac{g(\delta)}{(1 + n)^5 (1 - x)^3} \right] \sum_{f,g} a_f^\dagger a_f \\ - \frac{1}{2} \left[J_z \frac{(2 + x) g(\delta)}{(1 + n)^6 (1 - x)^4} + J_{xy} \frac{(1 + 3x)}{(1 + n)^4 (1 - x)^3} \right] \sum_{f,g} (a_f^\dagger b_g^\dagger + a_f b_g). \end{aligned} \tag{32}$$

The above Hamiltonian describes the anisotropic independent spin waves and can be diagonalized by means of (U, V) transformation. The anisotropic parameter η' can be determined by

$$\eta' = \frac{\left[\dfrac{(2+x)g(\delta)}{(1+n)^6(1-x)^4} + \dfrac{\eta_0(1+3x)}{(1+n)^4(1-x)^3}\right]}{\left[\dfrac{1+2x}{(1-x)^4(1+n)^5} + 4\dfrac{\eta_0 g(\delta)}{(1+n)^5(1-x)^3}\right]}. \tag{33}$$

where

$$\eta_0 = J_{xy}/J_z \tag{34}$$

$$x = g(\delta)^2/(1+n)^2. \tag{35}$$

We discuss the isotropic Heisenberg model where $\eta_0 = 1$. In conventional spin wave theory, the η in $\boldsymbol{H}_2(\eta)$ is set to one and the non-interacting representation is simplified. However, the excitation of the spin wave is still anisotropic (where $\eta' \approx 0.8309$) due to the remainder interaction including both the dynamics and kinematics. It means that the effective interaction along the Z direction is stronger than along the others and seems to imply the existence of symmetry breaking in the ground state. Our SCISWR is a self consistent independent spin wave ground state where the anisotropic parameter η must be given self-consistently by solving the fixed point of η^* in equation (33), where the quantities $(n, g(\delta), x)$ are related to the value of η through the definitions of (27), (30) and (35). A stable fixed point $\eta^* \approx 0.86004$ has been found by computer for 2D isotropic AFHM on a square lattice. $n \approx 0.07398$, $g(\delta) \approx 0.14773$ and the excitation energy of a spin wave in unrestricted Bose space is $\varepsilon_k^{(0)} = 0.6110J\sqrt{1-\eta^{*2}\gamma_k^2}$ with $\gamma_k = (\cos k_x d + \cos k_y d)/2$. In our SCISWR it is clear that

$$\langle\eta^*|\boldsymbol{H}_I(\eta^*)|\eta^*\rangle = 0. \tag{36}$$

Therefore, the Hamiltonian $\boldsymbol{H}$ is

$$\boldsymbol{H} = U_0 + \boldsymbol{H}_2(\eta^*) + \boldsymbol{H}_I(\eta^*). \tag{37}$$

Following the scheme of approximation in [12], the ground-state energy is

$$E = \sum_{f,g}\langle\eta^*|\boldsymbol{H}_{fg}\boldsymbol{P}|\psi\rangle/\langle\eta^*|\boldsymbol{P}|\psi\rangle \tag{38}$$

where $\boldsymbol{H}_{fg}$ is the pair Hamiltonian of two nearest-neighbour lattices $(\boldsymbol{f}, \boldsymbol{g})$, $|\eta^*\rangle$ is the ground state of $\boldsymbol{H}_2(\eta^*)$ and $|\psi\rangle$ is the exact ground state of the system.

As a first-order approximation, we neglect all correlations of sites $(\boldsymbol{f}, \boldsymbol{g})$ with others and the $|\psi\rangle$ in (38) is approximated by $|\eta^*\rangle$. The ground-state energy will be

$$E \approx \sum_{(f,g)}\frac{\langle\eta^*|\boldsymbol{H}_{fg}|\eta^*\rangle}{\langle\eta^*|\theta_f\theta_g|\eta^*\rangle}. \tag{39}$$

The calculation of equation (39) is similar to [14] and yields

$$\sum_{(f,g)}\langle\eta^*|\boldsymbol{H}_{fg}|\eta^*\rangle_{\eta^*} = -0.5J_z N\left[\frac{1+x}{1-x} + 4g\right]\frac{1}{[(1+n)^2-g^2]^2}$$

$$\langle\eta^*|\theta_f\theta_g|\eta^*\rangle_{\eta^*} = \left\{(1+2n)^2 + \frac{x}{1-x}\left[(1+2n)^2 - \frac{2(1+2n)}{1-x} + \frac{1+x}{(1-x)^2}\right]\right\}\Big/(1+n)^4. \tag{40}$$

Using the values of $n \approx 0.07398$, $g(\delta) \approx 0.14773$ and $x \approx 0.01892$ for 2D isotropic AFHM of square lattice, we have $E \approx -0.642JN$.

For a further approximation, we need to calculate the corrections from the correlation between sites (f, g) and others. Now, the ground-state energy is

$$\begin{aligned} E &\approx \sum_{\langle f,g\rangle} \frac{\langle\eta^*|H_{fg}|\eta^*\rangle}{\langle\eta^*|\theta_f\theta_g|\eta^*\rangle} \\ &\approx \sum_{\langle f,g\rangle} \frac{\langle\eta^*|H_{fg}\prod_i \theta_i|\eta^*\rangle}{\langle\eta^*|\theta_f\theta_g\prod_{i\neq f,g}\theta_i|\eta^*\rangle} \\ &\approx \sum_{\langle f,g\rangle} \frac{\langle\eta^*|H_{fg}|\eta^*\rangle}{\langle\eta^*|\theta_f\theta_g|\eta^*\rangle} \\ &+ 6[\Sigma_2 + \Sigma_1 + \Omega]/\langle\eta^*|\theta_f\theta_g\theta_{f_3}|\eta^*\rangle \\ &+ 8[\Sigma_2' + \Sigma_1' + \Omega']/\langle\eta^*|\theta_f\theta_{b_{fg}}\theta_{g_4}|\eta^*\rangle \\ &+ (\text{higher-order terms}) \end{aligned} \tag{41}$$

where the site f_3 is the nearest-neighbour of g, g_4 the next-nearest-neighbour of g, whereas Σ_1, Σ_2, Ω come from the corrections of nearest-neighbour correlation and Σ_1', Σ_2', Ω' from next-nearest-neighbour correlations. The term Σ_1 is from $-\langle\eta^*|\tilde{a}_f\tilde{b}_g\theta_{f_3}|\eta^*\rangle_c$ and $-\langle\eta^*|\tilde{a}_f^\dagger\tilde{b}_g^\dagger\theta_{f_3}|\eta^*\rangle_c$. The term Σ_1' is from $-\langle\eta^*|\tilde{a}_f\tilde{b}_g\theta_{g_4}|\eta^*\rangle_c$ and $-\langle\eta^*|\tilde{a}_f^\dagger\tilde{b}_g^\dagger\theta_{g_4}|\eta^*\rangle_c$, whereas Σ_2, Σ_2', Ω and Ω' are from $\langle\eta^*|S_f^z|\eta^*\rangle\langle\eta^*|S_g^z\theta_{f_3}|\eta^*\rangle_c$, $\langle\eta^*|S_f^z|\eta^*\rangle\langle\eta^*|S_g^z\theta_{g_4}|\eta^*\rangle_c$, $\langle\eta^*|S_f^zS_g^z\theta_{f_3}|\eta^*\rangle_c$ and $\langle\eta^*|S_f^zS_g^z\theta_{g_4}|\eta^*\rangle_c$, respectively, where $\langle\cdots\rangle_c$ means the average taken from linked diagrams. Meanwhile,

$$\langle\eta^*|\theta_f\theta_g\theta_{f_3}|\eta^*\rangle \approx \langle\eta^*|\theta|\eta^*\rangle^3 + 2\langle\eta^*|\theta|\eta^*\rangle\langle\eta^*|\theta_f\theta_g|\eta^*\rangle_c + \langle\eta^*|\theta|\eta^*\rangle\langle\eta^*|\theta_f\theta_{f_3}|\eta^*\rangle_c \tag{42}$$

and $\langle\eta^*|\theta_f\theta_g\theta_{g_4}|\eta^*\rangle$ can be obtained similarly. All averages $\langle\cdots\rangle$ can be calculated analytically without any approximation.

After the tedious calculation for all the terms in equation (38) by means of the Wick expansion theorem in the SCISWR, we obtain the ground-state energy

$$\begin{aligned} \langle H\rangle &\approx (-0.6420 - 0.01562 - 0.00153)NJ \\ &\approx -0.6592NJ \end{aligned} \tag{43}$$

where -0.01562 is from the nearest-neighbour correlations and -0.00153 from next-nearest-neighbour correlations. The value is quite close to the numerical calculations $-0.6692(8)$ [5, 6]. If we perform more higher-order approximations, we must consider the higher-order perturbation as well as higher-order correlations. Although we cannot show that all corrections from interactions will be convergent, every term in the expansion of perturbation, however, has been convergent since there is a gap in the boson excitation energy. Meanwhile, if we perform the higher-order perturbation calculation, the ground-state wavefunction will be

$$|\psi\rangle = |\eta^*\rangle + \sum_{n=1}^{\infty}\left[\frac{-Q}{H_2(\eta^*)}H_I(\eta^*)\right]^n|\eta^*\rangle \tag{44}$$

where Q is a projector operator to rule out the state $|\eta^*\rangle$. There are infinite diagrams contributing to $\langle 0|H_I|\psi\rangle$ even for second-order perturbation since our Hamiltonian has

infinite terms in remainder interactions. Fortunately, each line in a diagram will carry a small factor n (≈ 0.07398) or $g(\delta)$ (≈ 0.14773) and the lowest-order diagrams have at least four lines since we have used the SCISWR as a non-interaction representation. So, the term of higher-order perturbation could be expected to give higher-order contributions. The convergence of perturbation theory seems good. However, we cannot give a proof for the complete convergence of our perturbation series.

Finally, we must point out that the excitation in the momentum space $a_k^\dagger|\eta^*\rangle$ is not a real physical excitation. Its gap favourably overcomes the divergence in the perturbation series and seems to improve convergence. The real spin excitation must be in the proper space. Therefore, we must study the real physical spin excitation in proper space which is defined by the state $\tilde{a}_k^\dagger|\psi\rangle$ and find out whether it has a gap or not. For the isotropic Heisenberg model, we have

$$[\tilde{a}_f, H]_- = \tfrac{1}{2}J\sum_g[\theta_g\tilde{a}_f - \tilde{a}_f\tilde{b}_g^\dagger\tilde{b}_g] - \tfrac{1}{2}J\sum_g[\theta_f\tilde{b}_g^\dagger - \tilde{a}_f^\dagger\tilde{a}_f\tilde{b}_g^\dagger] \tag{45}$$

if we make some kind of approximation to reduce the first term on the right-hand side of (45) into

$$\tfrac{1}{2}J\sum_g[\theta_g\tilde{a}_f - \tilde{a}_f\tilde{b}_g^\dagger\tilde{b}_g] \approx A\sum_g\tilde{a}_f + B\sum_g\tilde{b}_g^\dagger. \tag{46}$$

where A and B are coefficients. A self-consistent approximation of an independent physical spin wave in proper space is an example of equation (46), but the approximation of equation (46) might be more general. Let us consider the following symmetry:

$$\sum_g[\theta_g\tilde{a}_f - \tilde{a}_f\tilde{b}_g^\dagger\tilde{b}_g]^\dagger_{\tilde{a}_f\leftrightarrow\tilde{b}_g^\dagger} \to \sum_g[\theta_f\tilde{b}_f^\dagger - \tilde{a}_f^\dagger\tilde{a}_f\tilde{b}_g^\dagger]. \tag{47}$$

In the same kind of approximation, the second term on the right-hand side of (45) will read

$$\tfrac{1}{2}J\sum_g[\theta_f\tilde{b}_g^\dagger - \tilde{a}_f^\dagger\tilde{a}_f\tilde{b}_g^\dagger] \approx A\sum_g\tilde{b}_g^\dagger + B\sum_g\tilde{a}_f. \tag{48}$$

Therefore, we obtain

$$[\tilde{a}_f, H]_- \approx (A-B)\sum_g\tilde{a}_f - (A-B)\sum_g\tilde{b}_g^\dagger. \tag{49}$$

It is reduced to the excitations without a gap.

In summary, we have applied our complete Bose transformation to the 2D isotropic antiferromagnetic Heisenberg model on a square lattice. An SCISWR has been introduced as a non-interaction representation instead of the conventional free spin wave representation (CFSWR). In SCISWR, the remainder interaction will be zero at first-order perturbation, but not in CFSWR. Meanwhile, our calculation shows that the excitation of an unrestricted boson is anisotropic and has a small gap. The gap can help one to overcome the divergence problem in any term of higher-order perturbation. The approximate ground-state energy has been calculated in SCISWR and the value is $-0.6592NJ$, which is quite close to the numerical value ($-0.6698NJ$). It is not proven that the energy of an unrestricted Boson excitation has no gap after the complete renormalization of the remainding interaction. An explanation for the possibility of isotropic excitation without a gap has been given for a boson restricted in proper space, but it is not rigorous.

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